On Some Iterative Method for Solving Nonlinear Equations

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Abstract— **We study an iterative differential-difference three-step method for solving system of nonlinear equations, which uses, instead of the Jacobian, the sum of derivate of differentiable parts of operator and divided difference of nondifferentiable parts. The numerical examples illustrate how the methods works.**

Keywords— Newton method; nonlinear system; nondifferentiability; secant method.

I. Introduction

More than tree hundred years have passed since a procedure for solving an algebraic equation was proposed be Newton in 1669 and later by Raphson in 1690[4].

The method is now called Newton's method or the Newton-Raphson method and is still a central technique for solving nonlinear equations. Many topic related to Newton's method still attract attention from researchers. The T. Yamamoto in his paper[1] described to trace historical developments in Kantorovich-type convergence theory as well as error estimates for Newton's method and Newton-like methods, mainly for differentiable equations in Banach spaces.

II. Problem

Consider the next problem

$$P(x)=0. \qquad (1)$$

Where P: D in X→Y. is continuous nonlinear operator defined on a nonempty convex subset D of Banach C with values in Banach space Y. This problem frequently occurs in numerical analysis. Many scientific and real-life problems can be formulates mathematically in terms of integral equations, boundary value problems, optimization, and so on. Generally, iterative methods along with their convergence analysis are used to find solutions of these equations. Many researchers [1-6] have extensively studied these problems and proposed many direct and iterative methods for their solutions along with their convergence analysis. The well-known quadratically convergent Newton's method[6] used for (1) is given by

$$x_{k+1}= x_k-[P'(x_k)]^{-1}P(x_k),\ k\geq 0, \qquad (2)$$

where x_0 in D.

This method and many others use the differentiability of P. Not much work is done by using nondifferentiability of P which can be express in the form

$$P(x)=F(x)+G(x)=0, \qquad (3)$$

where F, G: D in X→Y, are nonlinear operators, F is differentiable and G is continuous but nondifferentiable.

So in [2], Catinas consired

$$x_{k+1}= x_k-[A(x_k)]^{-1}P(x_k), \qquad (4)$$

$$A(x_k)= F'(x_k)+G(x_{k-1}, x_k),$$

And obtains superlinear convergence under certain conditions on the operators involed.

On [3], authors proposed next modification

$$y_k= \lambda x_k+(1- \lambda) x_{k-1},\ \lambda \text{ in } [0,1),$$

$$x_{k+1}= x_k-[F'(x_k)+G(y_k, x_k)]^{-1}P(x_k),\ k\geq 0$$

Two are the advantages: the first, the differentiable part of the operator is considered in the optimal situation; and the second, for the nondifferentiable part, the class of iterations is considered, which improves the result given by the Secant method. In their paper authors, gave semilocal convergence result when mild conditions are required.

S. Shakho with K. Argyros in [5] and the same method in [4] proposed two-step solvers for solving equations (3) defines as

$$x_{k+1}= x_k-[A(x_k)]^{-1}P(x_k),$$

$$y_{k+1}= x_{k+1}-[A(x_k)]^{-1}P(x_{k+1}),\ k\geq 0,$$

where x_0, y_0 in D are two starting points, and $A_k=[F'((x_k+y_k)/2)+G(y_k, x_k)]$. Their described local convergence analysis with super quadratic order.

III. Three-step method

In the present work, using the idea of construction of three-step method [6] and based on Steffensen's method, we propose the following method of solution of problem (3):

$$u_k= x_k-[A(x_k)]^{-1}P(x_k),$$

$$v_k= x_{k+1}-[A(x_k)]^{-1}P(x_k), \quad (5)$$

$$x_{k+1}=\arg\min\|P(u_k-\gamma(u_k- v_k))\|,\ k\geq 0.$$

An advantage of this method is that, in each iteration, it requires practically the same amount of calculation at the method (4) we perform only one additional one-dimensional minimization. And at the same time method (5) has a higher rate of convergence.

IV. Examples

The testing of the method was performed on the same examples.

Consider the system[7]

$$x^2-y+1+|x-1|/9=0,$$

$$y^2+x-7+|y|/9=0.$$

The solution of the system is $x^*=(1.15936;\ 2.36182)$.

TABLE I. Method (4) with $X^o=(0;0)$

n	x_n	y_n	$\|x_n-y_n\|$
1	1.376295	0.315782	1.412058
2	1.760693	2.209329	1.932169
3	1.254536	2.345947	0.524270
4	1.162582	2.361206	0.093211
5	1.159365	2.361824	0.003275
6	1.159361	2.361824	4.024E-6
7	1.159361	2.361824	6.08E-12

TABLE II. Method (5) with $X^o=(0;0)$

n	x_n	y_n	$\|x_n-y_n\|$
1	1.660855	2.300371	2.837278
2	1.213140	2.351301	0.450602
3	1.160459	2.361659	0.001105
4	1.159366	2.361824	5.38E-6
5	1.159361	2.361824	2.33E-9
6	1.159361	2.361824	6.08E-12

Consider the system[8]

$$3x^2y-+y^2-1+|x-1|=0,$$

$$y^4+xy^3-1+|y|=0.$$

The solution of the system is $x^*=(0.89465537;\ 0.3278265)$.

The method (4) didn't solution this system.

TABLE III. Method (5) with $X^o=(0.5;0.5)$

n	x_n	y_n	$\|x_n-y_n\|$
1	0.874583	0.386700	0.391343
2	0.902008	0.331289	0.008796
3	0.894424	0.326835	0.001066
4	0.894098	0.327722	0.000117
5	0.894652	0.327832	6.505E-6
6	0.894655	0.327827	6.062E-7
7	0.894655	0.327826	1.001E-7
8	0.894655	0.327826	2.166E-8
9	0.894655	0.327826	5.681E-9

The numerical results indicate that method (5) converges faster then method (4)

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