

Convective-thermal drying simulation to reduce the damage of the grain in convective plants

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Abstract – A physical-mathematical model is proposed for analyzing the process of drying the bed of one-type granular materials in a convective drying chamber under the action of a drying agent in order to optimize this process and reduce the percentage of grain damage. The basic relations of the coupled problems of mechano-diffusion and mass transfer for the considered grain are written, taking into account the heterogeneity of the grain structure (core and coat) and its position in the bed. The moisture concentration and the radial displacement vector are chosen as determining functions. On the basis of the obtained solutions of changes in moisture concentration in grain over time, taking into account the heterogeneity of the grain structure and the rate of blowing the drying agent for the intergranular medium, a numerical analysis of changes in moisture concentration, temperature, displacements and deformations in grain over time are carried out, taking into account taking into account the physical and chemical characteristics of the grain. It is found that the temperature of the drying agent has the greatest influence on drying (movement, deformation, grain stress).

Keywords – physico-mathematical model, drying process, granular materials, convective drying chamber, optimization, grain damage, interconnected equations

I. INTRODUCTION

Grains, by their structure, are capillary-porous colloidal bodies [1], in which the mechanism of mass transfer during dehydration is determined by the form and energy of moisture bonding with the material, its structure, moisture content, and drying conditions. The work [2] analyzes the factors determining the technological properties of grain during post-harvest processing. The physicochemical properties of grain depend on external conditions that change during drying, hydrothermal processing, and grain storage. Grain is a physiological culture consisting of an embryo, endosperm (kernel), and husks. The morphological structure (thickness of husks and aleurone layer), vitreosity, and chemical composition (content of proteins and starch) of wheat grain were studied in the work [3]. Internal moisture transfer in the grain is purely diffusive [4]. Water permeability changes the structural-mechanical properties of the endosperm, and contributes to the formation of microcracks, and its loosening. The difference in the structure of husks and endosperm affects not only the absorption or removal of moisture but also physicochemical processes that change the volume of grain and the structure of the endosperm,

therefore, it is necessary to account for this heterogeneity in the models describing processes of grain treatment. One more problem is that protein in grain at temperatures above permissible, undergoes denaturation, reducing its ability to swell, which is important for seed germination. Thus during drying it is important to prevent such a denaturation as well as traumas of the grain in general. According to scientists, each percentage of injuries in the seed material reduces its yield [5]. Therefore, mathematical modeling of grain drying processes in convective chambers is an urgent scientific task due to its practical importance in the agriculture and food industry. The model presented in this work is aimed at facilitating the development of efficient drying technologies, leading to improved grain preservation and reduced post-harvest losses. The main theoretical principles and methods of modeling convective drying are presented in the work [6].

II. FORMULATION OF THE PROBLEM AND GOVERNING SYSTEM OF EQUATIONS

We consider a layer of thickness L , which is composed of identical moist grains of radius R , and it is referred to the Cartesian coordinate system so that the Oz axis is perpendicular to its surfaces. Each grain is considered a two-layered sphere, so-called, of an equivalent volume, which is referred to a spherical coordinate system with the origin at its center ($r=0$). The drying process of the sphere occurs through its outer contact surface $r=R$ with the intergranular medium. The moisture concentration c_z at the location of the selected grain along the layer thickness z will be determined from solving the mass transfer problem in the intergranular space, where the air-vapor mixture is uniformly filtered in the mode of complete displacement.

We consider each sublayer of the sphere as a two-component solid solution consisting of particles of the main component (matrix) and water. Under isothermal conditions, the determining functions are taken to be the vector of radial displacements $u_r^{(i)}$ and the moisture concentration $c^{(i)}$. Then, for each individual sphere, these functions are found from the solution of the coupled equations of mechano-diffusion, which in the spherical coordinate system take the form:

$$\frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial (r^2 u_r^{(i)})}{\partial r} \right] - \xi^{(i)} \beta^{(i)} \frac{\partial c^{(i)}}{\partial r} = 0,$$

$$\frac{\partial (rc^{(i)})}{\partial \tau} = D^{(i)} \frac{\partial^2 (rc^{(i)})}{\partial r^2} - D_\varepsilon^{(i)} \frac{\partial^2 (r\varepsilon^{(i)})}{\partial r^2}, \quad (1)$$

where $\xi^{(i)} = 3K^{(i)} / (3K^{(i)} + 4G^{(i)})$ is the mechanical constant, $\varepsilon^{(i)} = \partial u_r^{(i)} / \partial r + 2u_r^{(i)} / r$; $D^{(i)}$ is the diffusion coefficient; $D_\varepsilon^{(i)}$ is the coefficient of the influence of the gradient of the volume deformation field on the mass flow; $K^{(i)}$ is the bulk modulus; $G^{(i)}$ is the shear modulus; $\beta^{(i)}$ is the concentration coefficient of volumetric expansion; τ is time. Here, the index $i=1$ corresponds to the inner area ($0 \leq r \leq r_*$), and $i=2$ corresponds to the outer area ($r_* \leq r \leq R$) of the sphere.

We assume that at the initial time ($\tau = 0$), radial displacements are zero, and concentrations are constant:

$$u_r^{(i)} = 0, \quad c^{(i)} = c_0^{(i)} \quad (i=1,2). \quad (2)$$

On the surface of the sphere $r = R$, which is free from external loads, the condition $\hat{\sigma}^{(2)} \cdot \vec{n} = 0$, where $\hat{\sigma}$ is the stress tensor, \vec{n} is the normal to the surface, is written as follows [7]:

$$\sigma_{rr}^{(2)} = \left(K^{(2)} + \frac{4}{3} G^{(2)} \right) \frac{\partial u_r^{(2)}}{\partial r} + 2 \left(K^{(2)} - \frac{2}{3} G^{(2)} \right) \frac{u_r^{(2)}}{r} - \beta^{(2)} K^{(2)} c^{(2)} = 0 \quad (3)$$

We assume that the density of the diffusion flow at the surface of the sphere $J_R^{(2)}$ is proportional to the difference in moisture concentrations at the surface of the grain $c_R^{(2)}$ and in the intergranular space c_z , i.e.,

$$-D^{(2)} \frac{\partial c^{(2)}}{\partial r} \Big|_{r=R} + D_\varepsilon^{(2)} \frac{\partial \varepsilon^{(2)}}{\partial r} \Big|_{r=R} = k (c^{(2)} - c_z). \quad (4)$$

Here, k is the mass transfer coefficient.

The moisture transfer equation in the intergranular space of the layer is as follows [8]:

$$\frac{\partial c_z}{\partial t} + v \frac{\partial c_z}{\partial z} = D_z \frac{\partial^2 c_z}{\partial z^2} + J, \quad (5)$$

where v is the velocity of movement of the air-vapor mixture in the layer; D_z is the moisture diffusion coefficient in the intergranular medium; $J = \alpha J_R$ is the intensity of the local moisture source due to evaporation from individual grains, α is a coefficient depending on the size of the grains (radius) and their packaging (cubic, volume-centered, or face-centered, etc.) [7].

The process of convective diffusion in the intergranular space of the layer, composed of identical grains, is much faster compared to the process of moisture diffusion from the volume of the grain to its surface. Therefore, we will further analyze the drying

process of the layer in a quasi-static approximation, i.e., we will determine the moisture concentration c_z from the solution of the equation

$$D_z \frac{d^2 c_z}{dz^2} - v \frac{dc_z}{dz} + J = 0 \quad (6)$$

in which the function J is known function of time. Eq. (6) is solved under the boundary conditions:

$$c_z \Big|_{z=0} = 0, \quad c_z \Big|_{z=L} = c_z^n. \quad (7)$$

Here c_z^n is the concentration of saturated vapor at the selected temperature.

III. ALGORITHM DEVELOPMENT FOR THE MOISTURE CONTENT DISTRIBUTION PROBLEM

Let us denote

$$\mathcal{G}^{(i)}(r, z, \tau) = c_z(z) - c^{(i)}(r, z, \tau). \quad (8)$$

Then the system (1) we write in the following form

$$\frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial (r^2 u_r^{(i)})}{\partial r} \right] = \frac{\partial \varepsilon^{(i)}}{\partial r}, \quad (1)$$

$$\frac{\partial \varepsilon^{(i)}}{\partial r} = -\xi^{(i)} \beta^{(i)} \frac{\partial \mathcal{G}^{(i)}}{\partial r}, \quad (9)$$

where $\tilde{D}^{(i)} = D^{(i)} - \xi^{(i)} \beta^{(i)} D_\varepsilon^{(i)}$ is an effective diffusion coefficient. In this case, the initial and boundary conditions take the form [7]

$$\mathcal{G}^{(i)} \Big|_{\tau=0} = c_z(z) - c^{(i)} \Big|_{\tau=0}, \quad \mathcal{G}^{(1)} \Big|_{r=0} \neq \infty, \quad \frac{\partial \mathcal{G}^{(1)}}{\partial r} \Big|_{r=0} = 0,$$

$$\frac{\partial \mathcal{G}^{(2)}}{\partial r} \Big|_{r=R} + H \mathcal{G}^{(2)} \Big|_{r=R} = 0. \quad (10)$$

Here $H = kR/D$.

Equality of flows at the interfaces of the sublayers of the grain $r = r_*$ gives us

$$\tilde{D}^{(1)} \frac{\partial \mathcal{G}^{(1)}}{\partial r} = \tilde{D}^{(2)} \frac{\partial \mathcal{G}^{(2)}}{\partial r}. \quad (11)$$

Solution of the boundary value problem (9)-(11) we search in the form:

$$r \mathcal{G}^{(i)} = \Theta^{(i)}(r) T^{(i)}(\tau). \quad (12)$$

For the functions $\Theta^{(i)}, T^{(i)}$, we have

$$\frac{d^2 \Theta^{(i)}}{dr^2} + \frac{\mu^2}{\tilde{D}^{(i)}} \Theta^{(i)} = 0, \quad \frac{dT^{(i)}}{d\tau} + \mu^2 T^{(i)} = 0, \quad (13)$$

where μ is the parameter of the problem.

The solutions to these equations are functions [7]

$$\Theta^{(i)} = \tilde{A}^{(i)} \cos(\lambda^{(i)} r) + \tilde{B}^{(i)} \sin(\lambda^{(i)} r),$$

$$T^{(i)}(\tau) = \tilde{K}^{(i)} e^{-\mu^2 \tau}, \quad (14)$$

which, using the dimensionless variable $\bar{r} = r/R$, can be written as [7]

$$\Theta^{(i)} = \tilde{A}^{(i)} \cos(\lambda_R^{(i)} \bar{r}) + \tilde{B}^{(i)} \sin(\lambda_R^{(i)} \bar{r}),$$

$$T^{(i)}(\tau) = \tilde{K}^{(i)} e^{-\mu^2 \tau}, \quad (15)$$

where $\tilde{A}^{(i)}, \tilde{B}^{(i)}, \tilde{K}^{(i)}$ are constants, $\lambda_j^{(i)} = \mu_j / \sqrt{\tilde{D}^{(i)}}$, $\lambda_R^{(i)} = \lambda^{(i)} R$.

For the region $0 \leq r < r_*$, taking into account the boundedness of the solution $\mathcal{G}^{(r,\tau)}$, if $r = 0$, we put $\tilde{A}^{(j)} = 0$ and we can write

$$\mathcal{G}^{(1)}(r, \tau) = \frac{1}{r} \sum_{j=1}^{\infty} B_j^{(1)} \sin(\lambda_R^{(1)} \bar{r}) e^{-\mu_j^2 \tau}, \quad (16)$$

while in the region $r_* \leq r \leq R$ we have [7]

$$\mathcal{G}^{(2)}(r, \tau) = \frac{1}{r} \left[A_j^{(2)} \cos(\lambda_j^{(2)} r) + B_j^{(2)} \sin(\lambda_j^{(2)} r) \right] e^{-\mu_j^2 \tau}. \quad (17)$$

Here $A_j^{(i)}, B_j^{(i)}$ are the coefficients of the expansion of $\tilde{A}^{(i)}, \tilde{B}^{(i)}$ in a series with respect to the eigenvalues μ_j of the corresponding characteristic equation obtained from the conditions (9), (10). We have

$$p_{11} A_j^{(2)} + p_{12} B_j^{(2)} = 0, p_{21} A_j^{(2)} + p_{22} B_j^{(2)} + a_{11} p_{23} B_j^{(1)} = 0, \\ p_{33} B_j^{(1)} - (c_z - c_0^{(1)}) = 0. \quad (18)$$

where c_0 is the concentration value $c^{(1)}$ for $r = 0, \tau = 0$.

$$p_{11} = a_{21} \cos \lambda_{Rj}^{(2)} + \lambda_{Rj}^{(2)} \sin \lambda_{Rj}^{(2)}, \\ p_{12} = a_{21} \sin \lambda_{Rj}^{(2)} - \lambda_{Rj}^{(2)} \cos \lambda_{Rj}^{(2)}, \\ p_{21} = \cos(\lambda_{Rj}^{(2)} \bar{r})_* + \lambda_{Rj}^{(2)} \bar{r}_* \sin(\lambda_{Rj}^{(2)} \bar{r}_*), \\ p_{22} = \sin \lambda_{Rj}^{(2)} \bar{r}_* - \lambda_{Rj}^{(2)} \bar{r}_* \cos \lambda_{Rj}^{(2)} \bar{r}_*, \\ p_{23} = -a_{11} \left[\sin(\lambda_{Rj}^{(1)} \bar{r})_* - \lambda_{Rj}^{(1)} \bar{r}_* \cos(\lambda_{Rj}^{(1)} \bar{r}_*) \right], \\ p_{33} = \lambda_j^{(1)}, \\ a_{21} = \left(-1 + \frac{kR}{\tilde{D}^{(2)}} \right), a_{11} = d^2, \\ \mu_j r_* / \sqrt{\tilde{D}^{(1)}} = d \lambda_{Rj}^{(2)} \bar{r}_*, \bar{r} = r / R, \\ \mu_j r_* / \sqrt{\tilde{D}} = \lambda_{Rj}^{(2)} \bar{r}_* \quad d = \sqrt{\tilde{D}}. \quad (19)$$

Determinant of the system (18) is

$$\Delta(\mu) = \begin{vmatrix} p_{11} & p_{12} & 0 \\ p_{21} & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{vmatrix} = \begin{vmatrix} p_{11} p_{12} \\ p_{21} p_{22} \end{vmatrix} p_{33}. \quad (20)$$

Expanding the determinant (20), we obtain the characteristic equation of the problem

$$t\mathcal{G} \left[\lambda_{Rj}^{(2)} (1 - \bar{r}_*) \right] = \lambda_{Rj}^{(2)} \frac{(1 - \bar{r}_*)}{a_{21} + (\lambda_{Rj}^{(2)})^2 \bar{r}_*}. \quad (21)$$

Let us note that as \bar{r}_* approaches zero, we obtain the characteristic equation [9] for a homogeneous sphere. By the roots λ_{Rj} of this equation, we find

$$\mu_j = \lambda_j^{(2)} \sqrt{\tilde{D}^{(2)}} \text{ and correspondingly } \lambda_j^{(1)}, \lambda_j^{(2)}.$$

The roots $\mu_1, \mu_2, \dots, \mu_j$ of the characteristic equation (21) are real values. To each root corresponds a solution of equations (16) and (17) as follows:

$$\mathcal{G}^{(i)}(r, \tau) = \frac{1}{r} \sum_{j=1}^{\infty} \Theta_j^{(i)} e^{-\mu_j^2 \tau},$$

where $A_j^{(1)} = 0, i = 1, 2; j = 1, 2, \dots, \infty$. (22)

In dimensionless form, we have:

$$\mathcal{G}^{(1)}(\bar{r}, \bar{z}) = \frac{1}{\bar{r}} \sum_{j=1}^{\infty} B_j^{(1)} \sin(\lambda_R^{(1)} \bar{r}),$$

$$B_j^{(1)} = \mathcal{G}^{(1)}(0, \bar{z}, 0) = (c_z(\bar{z}, 0) - c(0, \bar{z}, 0)) / \lambda_j^{(1)}.$$

$$\mathcal{G}^{(2)}(\bar{r}, 0) = \frac{1}{\bar{r}} \sum_{j=1}^{\infty} \left[A_j^{(2)} \cos(\lambda_{Rj}^{(2)} \bar{r}) + B_{Rj}^{(2)} \sin(\lambda_{Rj}^{(2)} \bar{r}) \right].$$

To determine $A_j^{(2)}, B_j^{(2)}, B_j^{(1)}$, let us denote by $\Delta_k(\mu_j) (k = \overline{1, 3})$ the algebraic adjunct Δ_{il} of the l^{th} element p_{il} of the i^{th} row of the determinant $\Delta(\mu_j)$, respectively. In particular,

$$\Delta_1(\mu_j, 1, \bar{r}_*) = f p_{12} p_{23}, \quad \Delta_2(\mu_j, 1, \bar{r}_*) = -f p_{11} p_{23},$$

$$\Delta_3(\mu_j, 1, \bar{r}_*) = f (p_{11} p_{22} - p_{21} p_{12}), \quad f = \mathcal{G}^{(1)}(0, \bar{z}, 0). \quad (23)$$

It is easy to see that the constants $A_j^{(2)}, B_j^{(2)}, B_j^{(1)}$ corresponding to the root μ_j are related to the algebraic complements by the relations:

$$\frac{A_j^{(2)}}{\Delta_1(\mu_j)} = \frac{B_j^{(2)}}{\Delta_2(\mu_j)} = \frac{B_j^{(1)}}{\Delta_3(\mu_j)} = M_j.$$

Hence

$$A_j^{(2)} = M_j \Delta_1(\mu_j), \quad B_j^{(2)} = M_j \Delta_2(\mu_j), \quad B_j^{(1)} = M_j \Delta_3(\mu_j)$$

Using the concept of orthogonality of eigenfunctions (22) for $\mathcal{G}^{(i)}(r, z, 0) = c_z(z, 0) - c_0^{(i)}(r, z, 0) = \text{const}$, we determine the coefficients M_j [10].

Using M_j defined by the relation (24), we can find $A_j^{(2)}, B_j^{(2)}, B_j^{(1)}$. The moisture concentration in an individual grain depending on its position coordinate \bar{z} within the layer is determined by the formula (8): $c^{(i)}(r, \bar{z}, \tau) = c_z(\bar{z}) - \mathcal{G}^{(i)}(r, \bar{z}, \tau)$. To determine $c_z(z)$, it is necessary to solve Eq. (6) with the corresponding boundary conditions.

IV. COMPUTER ANALYSIS OF CONVECTIVE DRYING

Based on these solutions, changes in moisture concentration in the grain depending on the air blowing velocity, the location of the grain in the layer, and the thickness of the wheat endosperm was studied. The following experimental data were taken into account for the calculation: moisture concentration at the initial time $c_0 = 5 \cdot 10^{-4}$ kg/m³; grain layer thickness $L = 1000$ mm; grain volume $V = 16.77$ mm³; effective moisture diffusion coefficient $\tilde{D}^{(2)} = 1.34 \cdot 10^{-3}$ mm²/s; reduced mass transfer coefficient $H = 10^{-7}$; equivalent grain radius $R = 2.00086$ mm; Fourier number $Fo = 2.00086 \cdot 10^{-7}$; diffusion coefficient $D_z = 10^2$ mm²/s; saturated vapor concentration $c_z^n = 1.34 \cdot 10^{-3}$; mechanical constant $\xi = 3K / (3K + G) = 0.5$; $\nu = 0.2$; $E = 16.4 \cdot 10^3$ g/mm²; volumetric expansion

$\beta = -2.71389 \cdot 10^{-4}$ mm/s; shear modulus
 $G = 6.8333 \cdot 10^3$ g/mm²; bulk modulus
 $K = 9.11111 \cdot 10^3$ g/mm²; the heterogeneity boundary
 $r_* = 0.9R$.

V. CONCLUSIONS

A physical-mathematical model is proposed for optimizing the drying process of homogeneous granular materials in a convective chamber to reduce grain damage. Relations for mechano-diffusion and mass transfer, considering grain structure heterogeneity, are outlined. Moisture concentration and radial displacement are key functions. Numerical analysis reveals a significant impact of the blowing rate of the drying agent on drying. Knowing the changes in moisture concentration and temperature, we can determine the stress-strain state of the grains, which allows for adjusting the drying process in convective-type drying plants to prevent grain cracking by not exceeding the grain's strength limit during drying.

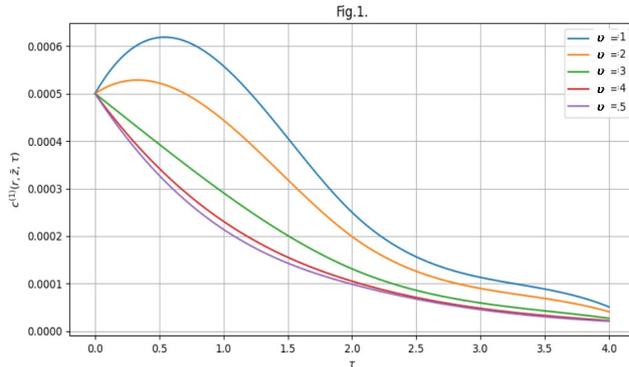


Fig. 1. Moisture concentration distribution over time in a grain's core $r=0.05R$ for different values of $\nu=1, 2, 3, 4, 5$ for $\bar{z}=0.5$

After the first hour of drying the grain with the rate of blowing $\nu=1$ and 2, we observe moisture accumulation inside the grains, the magnitude of which even exceeds the initial moisture content of the grain (Fig. 1, blue and orange curves). During two hours, there is rapid drying of the material, which continues into the third and fourth hours, but not as intensively. By the fifth hour of drying, it reaches a steady state.

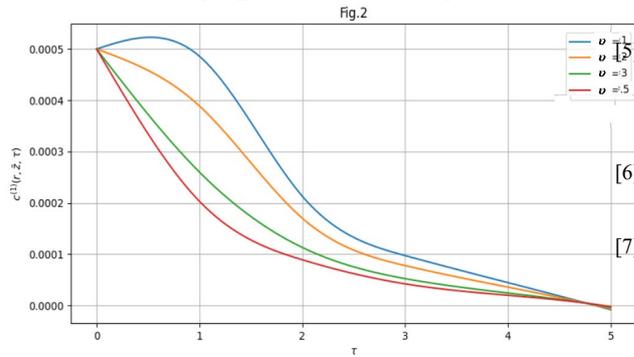


Fig. 2. Moisture concentration distribution over time in a grain's coat $r=0.95R$ for different values of values of $\nu=1, 2, 3, 4, 5$ for $\bar{z}=0.5$

In contrast, in the coat, moisture gradually begins to decrease after the first hour of the process for most rates of blowing (Fig. 2, orange, green and red curves). Further on, the drying processes in the core and the coat synchronize. The most effective drying of the grain occurs with the rates of blowing $\nu=4, 5$ (Fig. 1, 2 violet and red curves).

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