Simulation of Resonanse Oscillations of a Cantilever-fixed Polymer Rod

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Abstract— **The process of resonance oscillations of a cantilever-fixed polymer rod with a rectangular cross section is considered. The values of the resonant frequencies of the own oscillations of the rod were obtained. The possibility of determining the real and imaginary parts of a complex dynamic Young's modulus of a polymeric rod at basic resonance frequency is shown.**

Keywords— resonant vibrating-reed method; oscillation amplitude, Young's dynamic modulus

I. Introduction

Young's complex dynamic modulus (E^*) and the tangent of mechanical losses (*tgδ*) of a number of polymeric materials, the method of forced resonant oscillations of a cantilever-fixed sample was used as a rod of rectangular shape at sound frequencies [1].

The essence of the method is to measure the amplitude of the oscillation (*A*) of the free end of the rod when changing the frequency of the driving force applied to the other fixed end.

II. Vibrations Simulation and Resonance Frequency Determination

The behavior of a sample of a polymer material (fig. 1) during oscillations under the disturbing force is described by the following differential equation [2]:

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} + E^* \chi^2 \frac{\partial^4 u(x,t)}{\partial x^4} = 0 , \quad (1)$$

where $u(x,t)$ is function of the dependence of points transverse displacements of the rod axis on the coordinate x and time t; ρ is the density of the polymer material.

The solution of equation (1) is represented as a harmonic function

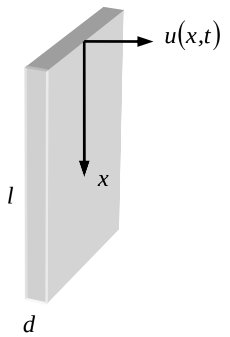

Figure 1. Specimen and its vibrating coordinate system

$$u(x,t) = X(x) e^{i\omega t} , \quad (2)$$

then, we get

$$\frac{d^4 X(x)}{dx^4} + k^* X(x) = 0 , \quad (3)$$

where k^* complex wave number of oscillations per bend; ω cyclic oscillation frequency.

The general solution of equation (3) is as follows

$$X(x) = A_1 \cos k^* x + A_2 \sin k^* x + A_3 chk^* x + A_4 shk^* x , \quad (4)$$

where A_i arbitrary constants.

The boundary conditions for our problem are as follows

$$X(0) = X'(0) = 0; \quad X''(l) = X'''(l) = 0. \quad (5)$$

where l the length of the sample.

The integral of equation (3) satisfying the conditions at the end $x = 0$ has the following form:

$$X(x)=\frac{1}{2}A_3\left(chk^*x-cos\,k^*x\right)+\frac{1}{2}A_4\left(shk^*x-sin\,k^*x\right), \quad (6)$$

The conditions at the end $x=l$ are expressed by the following equations

$$\left.\begin{aligned} A_3\left(chk^*l+cos\,k^*l\right)+A_4\left(shk^*l+sin\,k^*l\right)=0, \\ A_3\left(shk^*l-sin\,k^*l\right)+A_4\left(shk^*l+sin\,k^*l\right)=0, \end{aligned}\right\} \quad (7)$$

where

$$\left(chk^*l+cos\,k^*l\right)^2-\left(sh^2k^*l-sin^2k^*l\right)=0, \quad (8)$$

or

$$chk^*l\,cos\,k^*l+1=0. \quad (9)$$

Putting $k^*l=a+ib$ at the resonance of the rod in the conditions of $a=a_i; b=0$, enables to obtain a ratio for the sample amplitude oscillations [3]

$$X_i(x)=C\left(cha_ix-cos\,a_ix-\frac{\left(sha_ix-sin\,a_ix\right)^2}{cha_ix+cos\,a_ix}\right), \quad (10)$$

where a_i the roots of the equation (9), C some constant, which can be determined from the results of the experiment.

The values a of and b are defined as follows

$$a\cong\frac{\omega^{\frac{1}{2}}l}{\left(\frac{E\chi^2}{\rho}\right)^{\frac{1}{4}}};\ b\cong\frac{1}{4}\frac{\omega^{\frac{1}{2}}l}{\left(\frac{E\chi^2}{\rho}\right)^{\frac{1}{4}}}tg\delta, \quad (11)$$

moreover, for a rectangular rod $\chi=\frac{d}{\sqrt{12}}$, d is the thickness of the sample.

The first four roots of the equation

$$kl=1{,}8751;\quad 4{,}6941;\quad 7{,}8548;\quad 10{,}9965. \quad (12)$$

On fig. 2 presents the dependence of the normalized amplitude X on the length of the rod for four values of kl.

According to $|X|$ measurements of the sample of transverse oscillations at different frequencies, a resonance curve is constructed, which parameters are the frequency of oscillations (f) and the ratio of amplitudes ($|X|/|X_{\max}|$), where X_{max} is the maximum value of the amplitude corresponding to the principal resonant frequency (f_r). For f_r determine the width of the resonance curve (Δf_r) at the level $|X_{\max}|/\sqrt{2}$.

Resonant oscillation frequencies in experimental studies can be modified by the polymer form factor (l/d) of the sample.

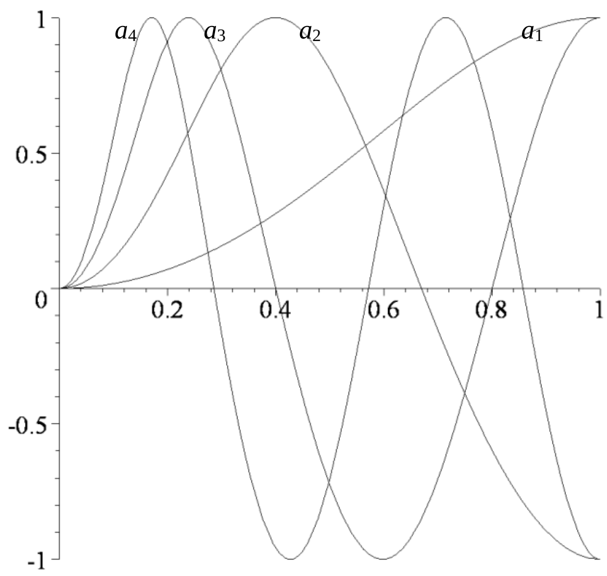

Figure 2. Normalized amplitude dependences for four values of a_i

In this case, at the fundamental resonant frequency (f_r,) which corresponds to the smallest root of equation (9), we obtain the following relation for E'

$$E'=\frac{48\pi^2\rho l^4 f_r^2}{1{,}8751^4 d^2}. \quad (13)$$

For the value of $tg\delta$ we have

$$tg\delta=\frac{\Delta f_r}{f_r}. \quad (14)$$

Accordingly, the imaginary part (E'') of complex E^* is defined as follows

$$E''=E'tg\delta. \quad (15)$$

In this case, we can determine the value of E^* in the following way:

$$E^*=\left(E'^2+E''^2\right)^{\frac{1}{2}}. \quad (16)$$

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