The Approach to the Solution Multi - Criteria Problems of Optimization

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Abstract— **The method for finding a compromise solving of multicriteria optimization problems with flexible limit constraints has been considered. The application of the method at simultaneous profit optimization and company's revenue has been regarded.**

Keywords— multicriteria optimization; technological matrix; compromise solving; flexible constraints; flexible planning.

I. Introduction

Depending on the relationship between alternative action plans and consequences, deterministic and non-deterministic decision-making problems are distinguished, and in terms of optimality - one-criterial and multi-criteria ones. In non-deterministic problems, some variables and parameters of an economic model are usually indeterminate, that is, for their values, only the intervals in which they can be are known. The exact values of such variables at the time of the decision making can not be uniquely established.

Uncertain factors may occur, in particular, in the following cases:

• People who do not pursue the same goals as those of their researcher may participate in the economic situation being investigated. For example, when planning a foreign trade of some state it is necessary to take into account the possible actions of other countries. It is often impossible to predict these actions.

• Uncertain factors may arise due to the uncertainty of some of the processes or variables. A typical example of such a factor is weather conditions. Therefore, such uncertainties are often called natural.

• Uncertain factors also often include parameters of the efficiency criterion (target functions), which is the evaluation of various impacts on the managed system if these parameters are not well known enough.

Let D be the domain of permissible solutions and $x \in D$.

In the above list of the most typical situations in the case of problems with uncertain factors, in the first place there is an impact on the situation of subjects that do not pursue the same goals as the researcher of the system.

In multicriteria optimization problems, there are several target functions $z_1 = f_1(x),\ z_2 = f_2(x), \dots,\ z_m = f_m(x)$, each of which can reach its maximum values at different points. In this case, the decision maker (PDM) must describe not only the domain of the permissible values D of the target function, but also specify the principle of choosing the final solution. Therefore, in the solution of multicriteria problems the role of the subjective factor, the role of knowledge and intuition of PDM increases in comparison with one-criterion problems.

As an example, let's consider the following problem [1]. Let the matrix A be the matrix of cost standards (technological matrix), Q - resource prices, P - prices of sales of products, B - reserves of resources. Then, if x units of production are planned, the cost of the necessary resources equals QAx, unpredictable revenue - Px, and the profit is also unpredictable and makes up $W = Px - QAx$ of the monetary units.

When solving such problems one can achieve simultaneous maximization of both revenue and profit. The optimization model of the formulated problem with two criteria will look like:

$$\begin{aligned} &Px \to \max, \\ &(P - QA)x \to \max, \end{aligned} \tag{1}$$

under conditions

$$Ax \le B, \quad x \ge 0.$$

Let's specify (1), taking in it

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 1 \end{pmatrix}, \quad Q = (1,\ 1,\ 4), \quad P = (17,\ 12), \quad B = \begin{pmatrix} 20 \\ 15 \\ 39 \end{pmatrix}.$$

Then in the expanded form the proposed model will look like:

$$z_1 = 17x_1 + 12x_2 \to \max, \tag{2}$$

$$z_2 = 3x_1 + 5x_2 \to \max, \tag{3}$$

$$\begin{aligned} &x_1 + 2x_2 \le 20, \\ &x_1 + x_2 \le 15, \\ &3x_1 + x_2 \le 39, \\ &x_1, x_2 \ge 0. \end{aligned} \tag{4}$$

It is easy to guess that the maximum value (2) under conditions (4) will be obtained at the point (12, 3). It is equivalent to $z_1^{\max}(12,\ 3) = 240.$ It is analogical for (3) – (4) $z_2^{\max}(10,\ 5) = 55.$

The final choice of the best solution for PDM.

In the proposed work, the choice of the best solution in multicriteria optimization problems is realized based on finding a compromise solution to the problem of linear programming with flexible threshold constraints.

II. Linear Vector-Optimization Model with Flexible Boundary Constraints

Let in some constraints

$$a_{i1}x_1 + ... + a_{in}x_n \le b_i \tag{5}$$

of the linear programming problems (LP) the limit b_i vary up to $b_i + d_i$, where $d_i \ge 0$, whereby different deviations from the value b_i are attributed to different limits of admissibility (the greater the deviation, the smaller the degree of its admissibility). This case is often encountered in practice. For example, the manufacturer is convinced that he needs to have the necessary raw materials b_i with high reliability and according to the supposed adjusted price. But he also believes that he needs to buy the next volumes of this raw material $b_i + d_i$, but without the guaranteed delivery of the surplus part, as well as its possible higher price.

Such a structure will be presented as follows

$$g_i(x) \equiv a_{i1}x_1 + ... + a_{in}x_n \le b_i,\ b_i + d_i, \tag{6}$$

where the "flexible" ratio «$\le$» should be interpreted as trying to surpass b_i, but remain in any case less than $b_i + d_i$».

Flexible relation (6) can be formalized based on constructing its membership function

$$\mu_i(g_i) : R \to [0,1]$$

with the following properties:

1) $\mu_i(g_i) = 1$ for $g_i \le b_i$,
2) $\mu_i(g_i) = 0$ for $g_i > b_i + d_i$,
3) $\mu_i(g_i) \in [0,1]$ for $b_i < g_i \le b_i + d_i$,
4) $\mu_i(g_i)$ monotonously falls on $[b_i,\ b_i + d_i]$.

The most used (with properties 1) - 4)) membership functions are linear and piecewise linear membership functions [2]. An analytic record of the simplest linear membership function is:

$$\mu(x) = \begin{cases} 1 & g \le b, \\ 1 - \dfrac{g-b}{d} & b < g \le b + d, \\ 0 & g > b + d. \end{cases} \tag{7}$$

Here, for the sake of simplicity, the index i is not used.

Let's consider the simplest Fuzzy-LP-Models

$$Z(x) = \begin{pmatrix} z_1(x) \\ ..... \\ z_k(x) \end{pmatrix} = \begin{pmatrix} c_1 \bullet x \\ .... \\ c_k \bullet x \end{pmatrix} \tag{8}$$

under conditions

$$g_i(x) \equiv a_i \cdot x \le b_i,\ b_i + d_i,\ \ i = \overline{1, m_1},$$
$$g_i(x) \equiv a_i \cdot x \le b_i,\ \ i = \overline{m_1 + 1,\ m},$$
$$x \ge 0$$

with really significant vectors

$$x = (x_1, ..., x_n),\ c_j = (c_{j1}, ..., c_{jn}),\ j = \overline{1, k},\ a_i = (a_{i1}, ..., a_{in}),\ i = \overline{1, m}$$

and actual values $b_i,\ i = \overline{1, m};\ d_i > 0,\ i = \overline{1, m_1}.$

In (8), the symbol «$\bullet$» means a scalar product.

Let's assume now that in the model (8) for each $i = \overline{1, m_1}$ there is a membership function $\mu_i(x)$ with the properties 1) - 4). For the defuzzification of the model (8) we shall consider μ_i as piecewise linear continuous functions. We introduce for (8) the notion of the set of admissible solutions (universal set):

$$X_u = \{x \in X_n^+ \mid g_i(x) < b_i + d_i\ \ \forall i = \overline{1, m_1}\ \text{ i } g_i(x) \le b_i\ \ \forall i = \overline{m_1 + 1,\ m}\},$$

where X_n^+ – an integral half-space of the Euclidean space R^n, and a set

$$X_s = \{x \in X_n^+ \mid g_i(x) \le b_i,\ i = \overline{1, m}\}.$$

To establish a meaningful compromise solution of the model (8) it is necessary to compare different target values $z_j(x)$, where $j = \overline{1, k}$.. To do this, first, it is necessary to find the optimal solution of the LP model

$$\max_{x \in X_u} z_j(x), \tag{9}$$

where $j = \overline{1, k}.$

The maximal values obtained here will be denoted as $\overline{z}_j = z_j(x_j^{**}),\ j = \overline{1, k}.$ Then the PDM chooses

$$\mu_{zi} = 1 \text{ for } z_j \ge \overline{z}_j,\ j = \overline{1, k}. \tag{10}$$

The lower bounds of the target functions are selected as follows:

$$\underline{z}_j = \min(\underline{z}_j^s, \underline{z}_j^u),\ \ j = \overline{1, k},$$

where

$$\underline{z}_j^u \min(z_j(x_1^{**}), ..., z_j(x_{j-1}^{**}), z_j(x_{j+1}^{**}), ..., z_j(x_k^{**}))$$

and

$$\underline{z}_j^s = \min(z_j(x_1^*), ..., z_j(x_{j-1}^*), z_j(x_{j+1}^*), ..., z_j(x_k^*)),$$

where $x_j^*,\ j=\overline{1,k}-$ optimal solution of the LP model (9).

For $z_j \le \underline{z}_j$ the equality is fulfilled $\mu_{z_j}=0,\ j=\overline{1,k}.$

As it is shown in [2, 3], optimizing, the system (8) is defuzzified in its equivalent

$$\lambda \to \max \tag{11}$$

under conditions

$$\lambda \le \mu_{z_j}(x),\ \ j=\overline{1,k},$$
$$\lambda \le \mu_i(x),\ \ i=\overline{1,m_1},$$
$$x \in X_u,\ \ \lambda \ge 0.$$

As it has been already described above, we approximate the membership functions $\mu_{z_j}(x),\ j=\overline{1,k}$ and $\mu_i(x),\ i=\overline{1,m_1}$, that are in (11), by continuous piecewise linear functions (in order for the model (11) to be linear). In addition, we will assume that PDM, knowing $\underline{z}_j$ and $\overline{z}_j,\ j=\overline{1,k},$ as well as $b_i, b_i+d_i,\ i=\overline{1,m_1},$ for each purpose and each exceedance of restrictions, indicates the level of requirements

$$z_j^A \in]\underline{z}_j, \overline{z}_j[\text{ i } g_i^A \in [b_i+d_j[. \tag{12}$$

If PDM is unable to select one or more of these requirements, we recommend that these values should be evaluated as follows:

$$z_j^A = \frac{\underline{z}_j+\overline{z}_j}{2} \text{ i } g_i^A = b_i + \frac{d_i}{2}. \tag{13}$$

Obviously, on such basis of PDM it is necessary to decide on x, when

$$\begin{aligned} & z_j(x) \ge z_j^A,\ \ j=\overline{1,k}, \\ & g_i(x) \le g_i^A,\ \ i=\overline{1,m_1}. \end{aligned} \tag{14}$$

Now taking into account the requirements (12) or (13) of the membership function that in (11) are determined according to the relations

$$\mu_i(x)=\begin{cases} 1 & g_i(x) \le b_j, \\ 1-\dfrac{g_i(x)-b_i}{g_i^A}\cdot(1-\lambda_A) & b_i \le g_i(x) \le g_i^A, \\ \dfrac{b_i+d_i-g_i(x)}{b_i+d_i-g_i^A}\cdot\lambda_A & g_i^A < b_i(x) \le b_i+d_i, \\ 0 & b_i+d_i < \mathrm{g_i(x)} \end{cases} \tag{15}$$

And, respectively,

$$\mu_{z_j}(x)=\begin{cases} \dfrac{z_j(x)-\underline{z}_j}{z_j^A-\underline{z}_j}\cdot\lambda_A & \underline{z}_j \le z_j(x) \le z_j^A, \\ \lambda_A+\dfrac{z_j(x)-z_j^A}{\overline{z}_j-z_j^A}\cdot(1-\lambda_A) & z_j^A < z_j(x) \le \overline{z}_j. \end{cases} \tag{16}$$

.

The formulas (15) - (16) define the equations of straight lines that pass through $(\underline{z}_j, 0)$, (z_j^A, λ_A), $(\overline{z}_j, 1)$ points із $j=\overline{1,k}$ and $(b_1, 1)$, $((g_i^A, \lambda_A)$, $(b_i+d_i, 0)$ із $i=\overline{1,m_1}$ and form broken lines.

If in this case we obtain the concave membership functions, i.e.

$$z_j^A > \lambda_A \overline{z}_j + (1-\lambda_A)\underline{z}_j \tag{17}$$

and

$$g_i^A > \lambda_A b_i + (1-\lambda_A) d_i, \tag{18}$$

then these functions by dividing the intervals $[\underline{z}_j, \overline{z}_j]$ and, accordingly, $[b_i, b_i+d_i]$ need to be reduced to $[b_i, b_i+d_i']$ where $d_i > d_i'$, so that in shorter intervals they become convex. In this case, the reduction of the interval is not sought after a compromise, because only such solutions as $\lambda \ge \lambda_A$ are taken into account. When $\lambda < \lambda_A$ as the value of target functions as those of restrictions are getting worse. After changes, unlike (15) - (16), we will get simpler relations

$$\mu_i(x)=\begin{cases} 1 & g_i(x) \le b_i, \\ 1-\dfrac{g_i(x)-b_i}{g_i^A-b_i}\cdot(1-\lambda_A) & b_i < g_i(x) \le \dfrac{g_s^A-\lambda_A b_s}{1-\lambda_A}, \\ 0 & \dfrac{g_i^A-\lambda_A b_i}{1-\lambda_A} < g_i(x), \end{cases} \tag{19}$$

and also

$$\mu_{z_i}(x)=\begin{cases} 0 & \underline{z}_\mathrm{j} \le \mathrm{z_j}(x) \le \dfrac{\mathrm{z_j^A}-\lambda_A \overline{\mathrm{z}}_\mathrm{j}}{1-\lambda_A}, \\ 1-\dfrac{\overline{z}_j-z_j(x)}{\overline{z}-z_j^A}\cdot(1-\lambda_A) & \dfrac{z_j^A-\lambda_A\overline{z}_j}{1-\lambda_A} < z_j(x) \le \overline{z}_j \end{cases} . \tag{20.}$$

If now, in coordinated intervals, all membership functions are convex and piecewise continuous, then the optimization model (11) is equivalent to the LP model in which membership functions μ_{z_i} and μ_i are written through (15) - (16) or (19) - (20).

In [3] we also mention the iterative MOLPAL algorithm. MOLPAL is a shortened sentence entry: Multi Objective Linear Programming based on Aspiration Levels.

Let's note that in the model (11), the entire utility value is determined through the parameter

$$\lambda(x) = \min(\mu_{z_1}(x), ..., \mu_{z_k}(x), \mu_1(x), ..., \mu_{m_1}(x)).$$

Let's determine

$$\max_{x \in X_u} \lambda(x),$$

i.e., the approach described here ensures that the guaranteed result is implemented. This principle ensures the choice of a guaranteed strategy, since it offers a judicious decision in the absence of information about the laws that govern the object being investigated and the logic of behavior of external entities. The application of these calculations provides caution in case of incomplete information. PDM can ignore the strategy outlined above when making a decision, i.e., to take risks. The method does not answer the question of how certain risky decisions will affect the outcome. It provides PDM with information about possible outcomes of well-considered actions. Only the one who is fully responsible takes the final decision.

III. CONCLUSION

In the proposed work, the choice of the best solution in multicriteria optimization problems is realized based on finding a compromise solution to the problem of linear programming with flexible threshold constraints. It should be noted that fuzzy restrictions can be given not in the form of intervals $[b_i, b_i + d_i]$ with a fixed lower limit, but may be based on knowledge of experts using fuzzy numbers. In addition, there are problems with an unclearly formulated goal function and vague parameters. In such cases, the application of the theory of fuzzy sets and fuzzy logical derivations [4] - [8], as well as fuzzy statistics [9] – [10] is considered.

The results of the computer simulation confirmed the validity of the described approach to finding a compromise solution of multicriteria problems with fuzzy data and conflicting objective functions.

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