Mathematical Modeling of Coupled Wagon Movement on a Railway Hump

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Abstract—**The efficiency of classification yards is largely determined by the process of controlled humping of railwagons. This paper proposes a mathematical model for calculating the group speed of several coupled wagons (cuts) as they move over a hump. The proposed model is formulated as a Cauchy problem for the Cauchy problem for a Riccati-type nonlinear differential equation, taking into account the basic and additional specific rolling resistance of the wagons, which depend on their mass, type, and operating conditions. Both the forward problem (determining the speed at the end of the hump for a given initial speed) and the inverse problem (determining the required initial speed to achieve a specified speed at the end of the hump) are considered. The proposed approach enables numerical experiments using Euler and Runge–Kutta methods. The results can be applied in the design and operation of modern automatic control systems at classification yards, contributing to increased productivity, energy efficiency, and operational safety.**

Keywords—Mathematical modeling, classification yard in railway, direct calculation problem, inverse calculation problem, specific train resistance, ordinary differential equation, Cauchy problem.

I. Introduction

Classification yards are among the most important elements of railway infrastructure, providing the redistribution of car flows and the formation of trains in line with transportation needs. One of the key stages in a yard's technological cycle is the humping of wagons over the classification hump. The accuracy with which this process is controlled determines the system's overall throughput and processing capacity, operational safety, and energy efficiency.

In recent decades, considerable attention has been devoted to developing mathematical models that describe car dynamics during humping. Such models make it possible to predict speed and account for the influence of car mass, type, and axle count, as well as track resistance. Most existing studies consider the dynamics of individual wagons; however, practical tasks often involve the descent of a group of coupled wagons, whose motion is of a more complex nature.

Of particular interest is solving both the forward and inverse calculation problems. The forward problem consists in determining the exit speed from the hump given specified car characteristics and the track profile. The inverse problem makes it possible to determine the required initial speed to achieve a prescribed outcome at the end of the hump. Solving such problems is of great practical significance, as it enables the development of automatic control algorithms and the optimization of classification-hump parameters.

Accordingly, the aim of this study is to construct and investigate a mathematical model of the motion of several coupled wagons over a classification hump, taking into account their key characteristics and resistance to motion. The proposed approach is based on numerical methods for solving differential equations and can serve as a foundation for further research into the automation of classification processes.

II. Problem Formulation

The problem considered is to determine the arrival speed at the end (or beginning) of the descending section of a classification hump for several coupled railwagons on $1520[mm]$ gauge track. The mass of each car, the car type, and the axle count (four-, six-, or eight-axle) are assumed to be known, and all wagons are on roller bearings. The classification hump is modeled as a sequence of straight segments with a known gradient specified in per mille (‰), characterizing the slope angle. Since the total length of the coupled wagons is small compared with the length of the descending hump, the motion of the cut is modeled as a material point located at the cut's center of mass.

It is necessary to describe a mathematical model for calculating the group speed of several coupled wagons on a classification yard, with a known initial speed.

III. Mathematical Model of Motion

Let data be given for N general-purpose freight wagons of $1520[mm]$ gauge on roller bearings, indexed by $j = 1,2,\dots,N$. The data consist of m_j -the gross mass of car j , and coefficients a_j , b_j , c_j , which are, respectively, the mass fractions of four-, six-, and eight-axle wagons in the consist [1,2], determined from Tables I and II.

TABLE I. COEFFICIENTS a_j, b_j, c_j, WHEN $m_j > 6000[KG]$

General purpose carriages on roller bearings	*Path characteristics*	a	b	c
4-axle	*Linked*	3.0	0.100	0.0025
6-axle		8.0	0.100	0.0025
8-axle		6.0	0.038	0.0021
4-axle	*Stickless*	3.0	0.090	0.0020
6-axle		8.0	0.080	0.0020
8-axle		6.0	0.026	0.0017

TABLE II. COEFFICIENTS a_j, b_j, c_j, WHEN $m_j \le 6000[KG]$

General purpose carriages on roller bearings	*Path characteristics*	a	b	c
4-axle	*Linked*	1	0.044	0.00024
6-axle		1	0.044	0.00024
8-axle		-	-	-
4-axle	*Stickless*	1	0.042	0.00016
6-axle		1	0.042	0.00016
8-axle		-	-	-

As a first approximation, we will neglect the effect of the oncoming airstream. In this case, only the basic and additional specific resistance forces act on a unit mass of the wagons; denote them by W_0 and $gI(x)$, respectively. Here $g = 9.81[m/sec^2]$ is the acceleration due to gravity, and $I(x)$ is the grade in per mille (‰), determined by the design of the descending hump and dependent on the track section, as indicated by the argument x.

Let the hump consist of n straight sections of lengths L_k, $k = 1,2,\dots,n$ (typically $n = 3$), with grades I_k. Then $I(x)$ can be represented as

$$I(x) = I_k, \quad \text{if} \quad \sum_{i=1}^{k-1} L_i \le x < \sum_{i=1}^{k} L_i.$$

Then the kinematic equation of motion of the cut on the k -th section can be written as:

$$\frac{dv}{dt} = W_0(v) + gI_k, \quad t \ge 0, \tag{1}$$

where t is time, v is the train's speed, and $W_0 = W_0(v)$ is the basic specific resistance to motion, which depends on the composition of the cut and its speed. According to [1], the formula for determining $W_0(v)$ is written as follows:

$$W_0(v) = \frac{1}{m} \sum_{j=1}^{N} m_j W_{0j},$$

where m_j and W_{0j} are, respectively, the gross mass and the basic specific resistance of the j -th car, and $m = \sum_{j=1}^{N} m_j$.

In accordance with [1,2], the specific resistance to motion W_{0j} depends nonlinearly on the speed v (in $[km/h]$) and is determined differently depending on the gross mass m_j. Specifically, for $m_j > 6000[kg]$

$$W_{0j} = g \cdot \left(0.7 + \frac{a_j + b_j v + c_j v^2}{m_j}\right),$$

where a_j, b_j, c_j are, respectively, the mass fractions of four-, six-, and eight-axle wagons in the consist, determined from Table I.

And at $m_j \le 6000[kg]$

$$W_{0j} = g \cdot (a_j + b_j v + c_j v^2),$$

where a_j, b_j, c_j are determined from Table II.

In accordance with the assumption that the k -th section of the classification hump is straight, each section can be represented by the interval $[L_{k-1}, L_k]$. We assume that at time $t = 0$ the consist begins to move with a known initial speed v_0, i.e.,

$$v(0) = v_0. \tag{2}$$

Let t_k denote the instant at which the motion transitions from the segment $[L_{k-1}, L_k]$ to the segment $[L_k, L_{k+1}]$. Then the following equality holds:

$$\int_0^{t_k} v(\tau)\, d\tau = \sum_{i=1}^{k} L_i. \tag{3}$$

Meanwhile, the time interval spent on the track segment $[L_{k-1}, L_k]$ is determined by the inequality

$$t_{k-1} \le t \le t_k, \tag{4}$$

where $t_0 = 0$.

IV. The dİrect problem

The direct problem of computing the train speed is as follows: to determine the speed of a consist of N wagons with known characteristics m_j, a_j, b_j, c_j given the known lengths and grades of the track sections. For this purpose, differential equation (1) subject to condition (2) is solved on the domain $[0,\ L_*]$, where

$$L_* = \sum_{i=1}^{n} L_i$$

From a mathematical standpoint, (1)–(2) is a Cauchy problem for a first-order nonlinear ordinary differential equation (a Riccati equation). In the general case this equation is not integrable in closed form (by quadratures), so numerical methods are used to solve it, for example the Euler or Runge–Kutta methods [3, pp. 63–78; 4, pp. 89–97].

V. The inverse problem

Of particular interest is the problem of determining the initial speed of the train so that it arrives at the end of the descending hump with a prescribed speed v_*. From a mathematical standpoint, this is an inverse Cauchy problem, and it can typically be solved by substituting t maps to $-t$, whereby the derivative $\frac{dv}{dt}$ is replaced with $-\frac{dv}{dt}$.

The domain of the problem is the interval $[0,\ L_*]$, represented as a sequence of segments

$$\left[\sum_{i=n-k+2}^{n} L_i\,,\ \sum_{i=n-k+1}^{n} L_i\right],$$

where $L_{n+1} = 0$.

The initial condition is then written as

$$v(0) = v_*.$$

And in determining the values of t_k in (4), the following formula will be used:

$$\int_0^{t_k} v(\tau)\, d\tau = \sum_{i=n+1-k}^{n} L_i.$$

VI. Conclusion

In the present study, a mathematical model is developed for the motion of a group of coupled railwagons during descent over a classification hump. Unlike many existing investigations that are limited to analyzing the motion of a single car, the proposed approach accounts for the specifics of group motion for wagons of different types and axle counts, enabling a more faithful representation of real operating conditions at a classification yard.

The proposed model is formulated as a Cauchy problem for the Cauchy problem for a nonlinear Riccati-type differential equation that incorporates both the basic and additional specific resistances to motion. These resistances are treated as functions of car speed and mass, which improves the accuracy of the dynamical description. Two problem settings are formulated and examined: the forward problem, which determines the final speed of the cut given a known initial speed, and the inverse problem, which determines the required initial speed to achieve a prescribed terminal speed at the end of the hump.

The practical significance of the study lies in the applicability of the developed model to analyzing the operation of classification yards, designing new humps and modernizing existing ones, as well as developing control algorithms for braking systems. The use of numerical solution methods (Euler, Runge–Kutta, etc.) provides sufficient flexibility and allows the analysis of various hump profiles and changing car characteristics.

The results of the study can help increase the throughput of classification yards, improve the safety of shunting operations, and reduce energy consumption. Future work includes extending the model to incorporate aerodynamic drag, the effects of crosswinds and headwinds, and the interaction of coupled wagons depending on their relative arrangement. Another relevant direction is integrating the developed model into simulation systems for classification-yard technological processes and applying it within intelligent decision-support systems.

References

[1] M. Khadjimuhametova, “Improvement of methods of calculation of key parameters of a hump yard for Uzbekistan,” International Journal of Advanced Science and Technology, vol. 29, no. 5, 2020.

[2] M. Khadjimuhametova, A. Merganov, and R. Egamberdiev, “An innovative method of designing the surface and elements of the hump profiles,” AIP Conference Proceedings, vol. 2432, p. 030046, 2022.

[3] Крижанівська Т.В., Бойцова І.А. Конспект лекцій з дисципліни "Чисельні методи". Одеса, 2013. – 152 с.

[4] "Чисельні методи: конспект лекцій " / уклад. Шебаніна О.В., Тищенко С.І., Хилько І.І., Пархоменко О.Ю., Крайній В.О. Миколаїв: МНАУ, 2024. – 100 с.