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Idempotent elements of the semigro up $\boldsymbol{BB_{XX}(DD)}$ *defined by semilattice of the class* $\sum_{22}(\boldsymbol{XX}, \mathbf{55})$

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Abstract. The article discusses complete semigroups of binary relations defined by semilattices of the class $\sum_2(X,5)$. It is described idempotent elements of complete semigroups of binary relations defined by semilattices of the class $\sum_2(X,5)$. In the case of finite semigroups, the formula for calculating idempotent elements is obtained. It is shown that the order of any subgroup $G_X(D,\varepsilon)$ of a semigroup $B_X(D)$ does not exceed 2.

Keywords: complete semigroup, semilattice, binary relation, idempotent element.

I. INTRODUCTION

In this paper, we consider complete semigroups of binary relations defined by complete *X*-semilattice of unions. It is known that the properties of the complete semigroups of unions are closely related to the properties of the semilattice by which this semigroup is defined. Because of this, we fix the diagram of semilattices (We denote this class of semilattices with the symbol $\sum_2(X,5)$) and study the properties of the corresponding semigroup. Using the quantities of reflections, we also obtained the formula for calculating the number of idempotent elements of finite semigroups.

II. THE THEORETICAL PART

Let X and $\sum_2(X,5)$ represent, respectively, any non-empty set and such a class of semilattices X of mutually isomorphic unions, each element of which is isomorphic to the upper semilattices $D_\zeta = \{\breve{D}_\zeta, Z_1, Z_2, Z_3, Z_4\}$, satisfying the following condition:

$$Z_4 < Z_3 < Z_1 < \breve{D}_\zeta,\ Z_4 < Z_2 < \breve{D}_\zeta,\ Z_1 \backslash Z_2 \neq \emptyset,\ Z_2 \backslash Z_1 \neq \emptyset, Z_3 \backslash Z_2 \neq \emptyset, Z_2 \backslash Z_3 \neq \emptyset \ldots (1)$$

The upper semilattice satisfying condition (1) is shown in Fig. 1.

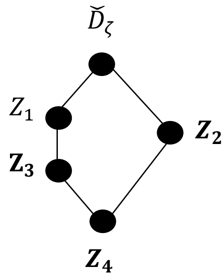

Fig:1

Theorem 1. Suppose $D \in \sum_2(X,5)$. A binary relation α of a semigroup $B_X(D)$ is an idempotent element of this semigroup only if it satisfies at least one of the following conditions:

a) $\alpha = X \times Z$ for any $Z \in D$;

b) α=(Y_1^α×Z)∪((X\Y_1^α)×Z'), where Z,Z'∈D, Y_1^α⊂X and satisfies the following conditions: Z⊂Z', Y_1^α≠∅, Y_1^α⊇Z და Z'\Y_1^α≠∅;

c) α =(Y_1^ α × Z)∪(Y_2^α×Z')∪((X\(Y_1^α∪Y_2^α))×Z") where Z,Z' ,$Z'' \in D$ and pairwise disjoint subsets Y_1^α and Y_2^α of set X satisfy the following conditions: Z ⊂ Z′ ⊂ Z'', $Y_1^\alpha \neq \emptyset$, $X \backslash (Y_1^\alpha \cup Y_2^\alpha) \neq \emptyset$, $Y_1^\alpha \supseteq Z$, $Y_1^\alpha \cup Y_2^\alpha \supseteq$ Z′, $Y_2^\alpha \cap$ Z′ $\neq \emptyset$;

d) α=(Y_1^α×Z_3)∪(Y_2^α×Z_2)∪((X\(Y_1^α∪Y_2^α))×D˘), where Y_1^α and Y_2^α are pairwise disjoint subsets of the set X that satisfy the following conditions: Z_3∩Z_2=∅, Z_3⊆Y_1^α⊆X\Z_2, Z_2⊆Y_2^α⊆X\Z_3;

e) $\alpha = (Y\_1^\alpha \times Z\_3\ ) \cup (Y\_2^\alpha \times Z\_2\ ) \cup ((X\backslash(Y\_1^\alpha \cup Y\_2^\alpha\ )) \times D˘\ )$, where Y_1^α and Y_2^α are pairwise disjoint subsets of the set X that satisfy the following conditions: $Z\_3 \cap Z\_2 = \emptyset, Z\_3 \subseteq Y\_1^\alpha \subseteq X\backslash Z\_2, Z\_2 \subseteq Y\_2^\alpha \subseteq X\backslash Z\_3;$

$$X\backslash(Y\_1^\alpha \cup Y\_2^\alpha\ ) \neq \emptyset, Y\_1^\alpha \supseteq Z, Y\_1^\alpha \cup Y\_2^\alpha \supseteq Z', Y\_2^\alpha \cap Z' \neq \emptyset;$$

where Y_1^α and Y_2^α are pairwise disjoint subsets of the set X that satisfy the following conditions: $Z_3 \cap Z_2 = \emptyset$, $Z_3 \subseteq Y_1^\alpha \subseteq X\backslash Z_2$, $Z_2 \subseteq Y_2^\alpha \subseteq X\backslash Z_3$;

a) $\alpha = (Y_1^\alpha \times Z_4) \cup (Y_2^\alpha \times Z_3) \cup (Y_3^\alpha \times Z_1) \cup \left((X\backslash(Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha)) \times \breve{D}\right)$, where Y_1^α, Y_2^α and Y_3^α are pairwise disjoint subsets of the set X that satisfy the following conditions:

$Y_1^\alpha \supseteq Z_4$, $Y_1^\alpha \cup Y_2^\alpha \supseteq Z_3$, $Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha \supseteq Z_1$, $Y_2^\alpha \cap Z_3 \neq \emptyset$, , $Y_3^\alpha \cap Z_1 \neq \emptyset$;

b) Let's say that $Z = Z_3$, $Z' = Z_2$ or $Z = Z_1$, $Z' = Z_2$, then

$\alpha = (Y_1^\alpha \times Z_4) \cup (Y_2^\alpha \times Z) \cup (Y_3^\alpha \times Z') \cup \left((X\backslash(Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha)) \times \breve{D}\right)$, where Y_1^α, Y_2^α and Y_3^α are pairwise disjoint subsets of the set X that satisfy the following conditions:

$Y_1^\alpha \supseteq Z_4$, $Y_1^\alpha \cup Y_2^\alpha \supseteq Z$, $Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha \supseteq Z'$, $Y_2^\alpha \cap Z \neq \emptyset$, $Y_3^\alpha \cap Z' \neq \emptyset$.

Theorem 2. Let *X be* a finite set and *I represent* the set of all idempotent relations of the semigroup $B_X(D)$. Then for the number $|I|$ the following statements are true:

a) if $\emptyset \notin D$ (i. e. $Z_4 \neq \emptyset$), then

$$|I| = 5 + \left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash Z_3|} + \left(2^{|Z_2\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash Z_2|} + \left(2^{|Z_1\backslash Z_4|} + 2^{|Z_1\backslash Z_3|} - 2\right) \cdot 2^{|X\backslash Z_1|} + \left(2^{|\breve{D}\backslash Z_4|} + 2^{|\breve{D}\backslash Z_3|} + 2^{|\breve{D}\backslash Z_2|} + 2^{|\breve{D}\backslash Z_1|} - 4\right)2^{|X\backslash \breve{D}|} + \left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot \left(3^{|Z_1\backslash Z_3|} - 2^{|Z_1\backslash Z_3|}\right)3^{|X\backslash Z_1|} + (\left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_3|} - 2^{|\breve{D}\backslash Z_3|}\right) + \left(2^{|Z_2\backslash Z_4|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_2|} - 2^{|\breve{D}\backslash Z_2|}\right) + \left(2^{|Z_1\backslash Z_4|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_1|} - 2^{|\breve{D}\backslash Z_1|}\right) + \left(2^{|Z_1\backslash Z_3|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_1|} - 2^{|\breve{D}\backslash Z_1|}\right)) \cdot\ 3^{|X\backslash \breve{D}|} + \left(2^{|Z_3\backslash Z_4|} - 1\right)\left(3^{|Z_1\backslash Z_3|} - 2^{|Z_1\backslash Z_3|}\right) \cdot \left(4^{|\breve{D}\backslash Z_1|} - 3^{|\breve{D}\backslash Z_1|}\right)4^{|X\backslash \breve{D}|} + \left(\left(2^{|Z_3\backslash Z_2|} - 1\right) \cdot \left(2^{|Z_2\backslash Z_3|} - 1\right) + \left(2^{|Z_1\backslash Z_2|} - 1\right) \cdot \left(2^{|Z_2\backslash Z_1|} - 1\right)\right) \cdot 4^{|X\backslash \breve{D}|}.$$

b) if $\emptyset \in D$ (i. e. $Z_4 = \emptyset$), then

$$|I| = 1 + \left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash Z_3|} + \left(2^{|Z_2\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash Z_2|} + \left(2^{|Z_1\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash Z_1|} + \left(2^{|\breve{D}\backslash Z_4|} - 1\right) \cdot 2^{|X\backslash \breve{D}|} + \left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot \left(3^{|Z_1\backslash Z_3|} - 2^{|Z_1\backslash Z_3|}\right) \cdot 3^{|X\backslash Z_1|} + (\left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_3|} - 2^{|\breve{D}\backslash Z_3|}\right) + \left(2^{|Z_2\backslash Z_4|} - 1\right) \cdot \left(3^{|\breve{D}\backslash Z_2|} - 2^{|\breve{D}\backslash Z_2|}\right)\left(2^{|Z_1\backslash Z_4|} - 1\right) \cdot (3\hat{\ }|D˘\backslash Z\_1\ | - 2\hat{\ }|D˘\backslash Z\_1\ |\ )) \cdot\ 3\hat{\ }|X\backslash D˘\ | + \left(2^{|Z_3\backslash Z_4|} - 1\right) \cdot \left(3^{|Z_1\backslash Z_3|} - 2^{|Z_1\backslash Z_3|}\right) \cdot \left(4^{|\breve{D}\backslash Z_1|} - 3^{|\breve{D}\backslash Z_1|}\right) \cdot 4^{|X\backslash \breve{D}|} + \left(\left(2^{|Z_3\backslash Z_2|} - 1\right) \cdot \left(2^{|Z_2\backslash Z_3|} - 1\right) + \left(2^{|Z_1\backslash Z_2|} - 1\right) \cdot \left(2^{|Z_2\backslash Z_1|} - 1\right)\right) \cdot 4^{|X\backslash \breve{D}|}.$$

Let us introduce the notation: let $G_X(D, \varepsilon)$ denote the maximal subgroup of the semigroup $B_X(D)$, which identity element ε represents the idempotent binary relation of the semigroup $B_X(D)$.

Theorem 3. For any idempotent binary relation ε of the semigroup $B_X(D)$, the subgroup $G_X(D, \varepsilon)$ of the semigroup $B_X(D)$ is a group which order does not exceed two.

CONCLUSION

From the above-mentioned we may conclude that digital technologies have brought changes to the nature and scope of education. Given that the integration of digital technologies is a complex and continuous process that impacts different actors within the education ecosystem, there is a need to show how these impacts are interconnected and identify the factors that can encourage an effective and efficient change in the education environments. The greatest difficulty is the transition from information circulating in the training system to independent practical actions and deeds, in other words, from a sign system as a form of information presentation on the pages of a textbook, a monitor screen, etc. to a system of practical actions performed on the basis of knowledge and having a fundamentally different logic than the logic of organizing a semiotic system. This is the classic problem of applying knowledge in practice, and in axiological language - the problem of the transition from worldview thought to relevant valuable action.

References:

1. Adriaans Pieter, Information, Stanford Encyclopedia of Philosophy, First published February 26, 2012; Substantive revision November 1, 2023. - URL:

https://plato.stanford.edu/entries/information/

2. Bagrationi Irma, For the Ethical Problems of Digital Education in Decision-Making Process, Proceedings of International Scientific Conference: PDMU-2023-XXXVIII - "Problems of Decision Making under Uncertainties", Kiev: "Видавництво Людмила", 2023. pp. 16-18. - URL: http://www.pdmu.univ.kiev.ua/PDMU_2023/PDMU-2023_End.pdf

3. Didmanidze I., Bagrationi I., Ulanov V., Matrosova N., Chargazia G., The Ethical Transformations of the Technological Systems For Digital Education Management, Proceedings of the conference: DTMIS'2020_Digital Transformation on Manufacturing, Infrastructure and Service", New York, 2021. - Article № 69, pp. 1-7. DOI: https://doi.org/10.1145/3446434.3446456

4. Didmanidze I., Bagrationi I., the Issue of Student Distance Communication and Collaboration, the Journal "Cross-Cultural Studies: Education and Science (CCS&ES)", Volume 3, Issue I, Vermont, 2018, Pp. 6-19. - URL: http://j-ccses.org/wp-content/uploads/2018/06/Issue-1-2018.pdf

5. Murrell Paul, Introduction to Data Technologies, First Edition, UK: "Chapman & Hall", 2009. - URL: https://www.stat.auckland.ac.nz/~paul/ItDT/itdt-2019-03-06.pdf