

# Mathematical Modeling of Current Leakage in the Combined Power Network of a Mine

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**Abstract**—Mathematical model of the cable line as a part of the combined mine power network was developed, taking into account the current leakage through the human body. Simulation has shown that the cable branch of the frequency converter is characterized by an unacceptably high probability of fatal electric shock.

**Keywords**—insulation; cable; leakage current; frequency converter; graph; matrix; ventricular fibrillation; model.

## I. INTRODUCTION

To date, most of the low-voltage power networks of coal mines are of the combined type because they are equipped with frequency converters (FC). The utilization of the latter makes it possible to adjust the rotation speed of the electric drives in a wide range. For instance, the UDK400, KDK500 coal shearers are equipped with frequency-controlled drive. Frequency converters PChV-250U5 are used as part of the drive of underground lifting machines. Due to the harsh operating conditions of underground electrical equipment, the probability of current leakage through the human body is high. To protect against leakages, the low-voltage power networks of the mine are equipped with a protective device. However, a leakage current to the ground in the cable, that connects the motor to the FC, cannot be detected by such a device with satisfactory reliability [1]. This determines the relevance of the scientific and technical problem of increasing the electrical safety of underground combined power networks.

The insulation capacitance is often considered lumped while studying the ground fault in cable lines [2]. The discrete nature of the FC output voltage is commonly ignored [3]. Most often, telegraph equations are used to study wave processes in long cable lines, which include partial derivatives with respect to the geometric parameters of the cable. To solve such equations, the finite element method for a specific point in time is used [4]. The scope of this approach is limited by the case of phase symmetry of the elementary sections resistances. This does not allow to correctly analyze the asymmetry that complicates the single-phase leakage of current to the ground through the human body. When analyzing the effects of electrocution on humans, it is necessary to take into account the frequency of leakage current [5].

Thus, the known mathematical models do not provide the necessary accuracy due to neglect of significant factors. However, known studies are the basis for the substantiation of

a mathematical model that takes into account wave processes in a long cable in case of asymmetrical current leakage.

The aim of study is to substantiate a mathematical model of the FC cable branch as part of a combined mine power network in event of leakage current through the human body.

## II. MATHEMATICAL MODEL OF CABLE BRANCH OF THE COMBINED POWER NETWORK

The cable line, that connects the motor to FC, is represented by the set of  $N$  elementary three-phase sections  $K_j, j=1 \dots N$ , of  $\Delta l$  length. An equivalent circuit of each section (Fig. 1) takes into account the resistances ( $R_{c\zeta j}$ ) and inductances ( $L_{c\zeta j}$ ) of the conductors, as well as the resistances ( $R_{g\zeta j}$ ) and capacitances ( $C_{g\zeta j}$ ) of the cable section insulation, and  $\zeta=\{a, b, c\}$  is the phase designation. The given equivalent circuit allows to investigate the mode of single-phase current leakage at a certain point of cable by reducing the insulation resistance of the corresponding section [6]. The node numbers of the circuit are set in accordance with the section number  $j$ .

An equivalent circuit of the cable line corresponds to Fig. 2 in the event that FC output voltages are matched sources  $u_a, u_b, u_c$  and active-inductive load is represented by  $R_{l\zeta}$  and  $L_{l\zeta}$  elements. A matrix-topological method is proposed to use for analyzing such equivalent circuit. The method involves the construction of a graph and the formation of a matrix  $\mathbf{F}$  of the main sections [7], the size of the latter depends on the number  $N$  of cable sections.

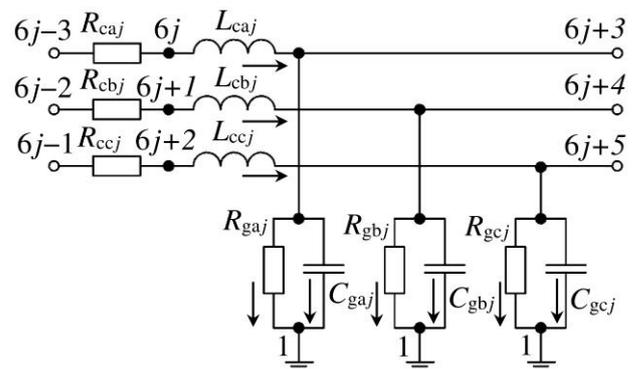


Figure 1. Equivalent circuit of the three-phase cable section  $K_j$

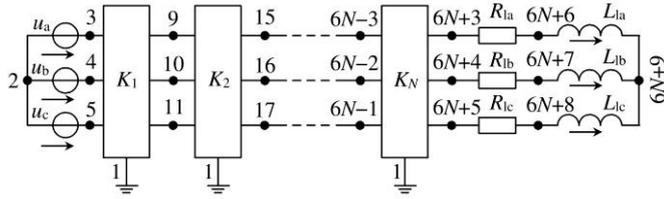


Figure 2. Equivalent circuit of the cable line as a part of combined power network

On the basis of matrix algebraic equations, compiled using the  $\mathbf{F}$  matrix in accordance with Kirchhoff's laws, as well as matrix differential equations with respect to derivatives of state variables, the matrix differential equation of state of the cable branch of the combined power network is obtained:

$$d\mathbf{X}/dt = \mathbf{B}_1^{-1} \cdot (\mathbf{B}_2 \cdot \mathbf{I}_R + \mathbf{B}_3 \cdot \mathbf{X} + \mathbf{B}_4 \cdot \mathbf{U}), \quad (1)$$

where  $\mathbf{X} = [\mathbf{U}_{ced} \ \mathbf{I}_{lch}]^T$  – vector of state variables;  $\mathbf{B}_1$ – $\mathbf{B}_4$  – matrix coefficients that are:

$$\mathbf{B}_1 = \text{diag} \left\{ \mathbf{C}_{ced} \quad \mathbf{L}_{ch} + \mathbf{F}_8^T \cdot \mathbf{L}_{ed} \cdot \mathbf{F}_8 \right\}; \quad (2)$$

$$\mathbf{B}_2 = \begin{bmatrix} \mathbf{Z} & -\mathbf{F}_3 \\ \mathbf{F}_6^T \cdot \mathbf{R}_{ced} & \mathbf{Z} \end{bmatrix}; \quad \mathbf{B}_3 = \begin{bmatrix} \mathbf{Z} & -\mathbf{F}_4 \\ \mathbf{F}_4 & \mathbf{Z} \end{bmatrix}; \quad \mathbf{B}_4 = \begin{bmatrix} \mathbf{Z} \\ \mathbf{F}_2^T \end{bmatrix}, \quad (3)$$

and  $\mathbf{Z}$  – zero matrix;  $\mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_6, \mathbf{F}_8$  – sub-matrices of  $\mathbf{F}$ .

A fragment of the combined 660 V mine power network is numerically analyzed by Simulink. The fragment includes Danfoss VLT Drive FC-302 N400T7 frequency converter of a 400 kW power output as a part of PRCh-400M unit, BiTmining NSSHCOEU 3x150+3x70/3 cable of 300 m length, and VAO5P560S6 induction motor of 400 kW rated power. The cable line was divided into  $N=100$  elementary sections. The frequency of the inverter output voltage is 50 Hz, the pulse-width modulation reference frequency is 5 kHz. The case of single-phase current leakage in the elementary section No. 50 of the cable through a human body with a resistance of 1 k $\Omega$  is considered. The trapezoidal interpolation method (ode23t solver) is used for numerically solving of (1). The integration step does not exceed  $1 \cdot 10^{-5}$  s. Simulation duration is 50 ms.

### III. SIMULATIONS RESULTS

A graph of instantaneous values of the leakage current through the human body was obtained in case of the emergency at the simulation time point of 11 ms (Fig. 3).

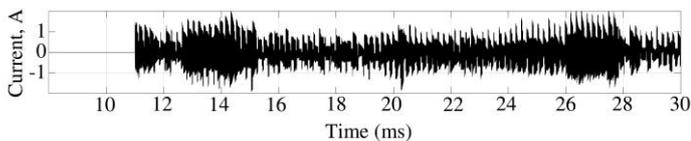


Figure 3. Graph of instantaneous values of leakage current through the human body as a function of time

The current is characterized by a polyharmonic composition and consists of fragments, each of which corresponds to the conduction intervals of power keys pairs of the FC autonomous voltage inverter. The peak values of the current through the human body reach 2 A. According to the obtained data and the method for calculating the equivalent current  $I_B$  value of the industrial frequency through the human body (IEC 60479-2), the estimate  $I_B = 0.51$  A. For the obtained  $I_B$  value if tripping time  $t_B < 200$  ms, the probability  $P_{vf}$  of ventricular fibrillation is about 0.05. At  $500 \text{ ms} < t_B < 2000$  ms, the probability  $P_{vf} \approx 0.5$ . If the time  $t_B > 5000$  ms, then  $P_{vf} > 0.5$ .

### IV. CONCLUSION

The obtained results suggest that the occurrence of current leakage through the human body in the FC cable branch of the combined power network is characterized by an unacceptably high probability of electrocution. The calculated value 0.05 of ventricular fibrillation probability with a current duration of up to 200 ms significantly exceeds the maximum permissible probability ( $1 \cdot 10^{-6}$ ) of the specified state. Such circumstance emphasizes the necessity of increasing the electrical safety of underground electrical power networks with isolated neutrals equipped with frequency converters.

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