On the Advanced Technique of Mathematical and Computer Simulation of Filtration Processes in Porous Media under the Influence of Physical and Chemical Factors

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Abstract— **The proposed improvement of the technique for deriving filtration equations in soils with variable porosity is described. The technique is based on the application of the apparatus of complete derivatives in the soil phase continuity equations. Examples of filtration equations in the case of a deformable and non-deformable skeleton of a porous medium are given.**

Keywords— mathematical model, purification, filtration, impurities, magnetic water purification, biological water purification, water lighting.

I. INTRODUCTION

Natural and artificial porous media are at least the three-component “solid-liquid-gaseous” components. Both solid and liquid components are able contain various chemical admixtures. The research of physical and chemical processes in such media requires taking into account their (processes) of interplays and interdependencies and consideration of porous medium as a complex heterogeneous system.

In the case of research of soil, as multicomponent porous media, several interconnected processes, the methodological aspects of building appropriate mathematical models play a crucial role. The interconnection of processes in heterogeneous porous media can be manifested in two aspects. The first is the interdependencies and interplays of flows [1], which are determined by the functions-factors of the process (pressure, temperature, concentration of chemicals, etc.). As an example, chemical osmosis, thermal osmosis, effect of thermal diffusion of soluted substances and so on. It is mathematically reflected in the modification of classical laws for porous media (e.g., Darcy’s, Fourier’s, and Fick’s laws). This approach to simulation of filtration processes in porous media, taking into account modifications of the Darcy’s, Fourier’s, Fick’s laws, was used, for example, in the works [1, 2, 4]. Secondly, due to interdependence of the parameters of the porous medium (porosity, porosity coefficient, filtration coefficient, etc.) and the determinative parameters of the process (temperature, concentration of chemicals, pressure in the porous liquid). Thus, the concentration of chemicals and temperature significantly affect the filtration coefficient of the porous medium [1, 3]. Similarly, dissolution of water-soluble components in a porous medium (chemical suffosion) [1-5] affects the porosity of the medium. In a similar manner, not only chemical suffosion, but also mechanical affect soil porosity. The Kozeny-Karman formula, in its turn, reflects dependence of the soil filtration coefficient on porosity.

At present, during research of several interconnected processes in saturated porous media the mathematical models and dependencies have been deducted so that, when a new factor of influence arises, practically all conclusions must be repeated again. Therefore, it is important to improve the methodology of studying interconnected processes in porous media, which would be automatically taken into account in the case of a new influence factor arising without repeating a number of previous judgments.

For concretization, let us consider the proposed advanced technique as an example of filtration processes in soil under conditions of variable porosity. Attention to change of porosity (or porosity coefficient) in a porous medium is reasonable. However, exactly porosity is one of the determinative characteristics of a porous medium that distinguishes its (medium) from a continuous medium. Change of porosity over time can mean compaction (consolidation) or swelling of the medium, thermal expansion (compression), chemical or (and) mechanical suffosion, availability of filtration deformations of the porous medium and so on.

II. CONTINUITY EQUATION OF LIQUID AND SOLID SOIL COMPONENTS USING TOTAL DERIVATIVE OVER TIME

The proposed improvements of the mathematical simulation technique are in taking into account the influence of factors on the parameters of the porous medium in an implicit form, using the apparatus of derivatives from complex functions derivation of equations. It allows to use the known formulas for parameters, or to use those formulas that will be offered later in the future. The fundamental form of the obtained equations of mathematical models will not change from it, and they (equations) will simply be supplemented by new experimental or theoretical data.

The basic equations of filtration processes in soil under conditions of variable porosity one can deduct based on the following dependencies:

- continuity equation of the liquid soil component

$$\nabla\cdot\left(\rho_p\,\mathbf{u}\right)+\frac{d\left(\sigma\rho_p\right)}{dt}=0; \qquad (1)$$

- continuity equation of the liquid soil component

$$\nabla\cdot\left(\rho_m\,\mathbf{v}\right)+\frac{d\left(\sigma_m\,\rho_m\right)}{dt}=0; \qquad (2)$$

where $\mathbf{u}$ - is the vector of filtration rate; $\mathbf{v}$ - is the vector of rate soil solid particles movement, which form its framework (they are not associated with porous liquid); σ - is soil porosity; σ_m - is the relative content of solid particles in the unit of porous medium volume; ρ_p - is density of porous liquid; ρ_m - is density of the soil solid particles material (including water-solluble and insoluble components); t - is time.

Availability of the total derivative over time means, for example

$$\frac{d\left(\sigma\rho_p\right)}{dt}=\frac{\partial\left(\sigma\rho_p\right)}{\partial t}+\sum_{i=1}^{n}\frac{\partial\left(\sigma\rho_p\right)}{\partial s_i}\cdot\frac{\partial s_i}{\partial t}, \qquad (3)$$

where $\mathbf{S}=\left(s_1,s_2,...,s_n\right)$; s_i, $i=\overline{1,n}$, characterize that or other factor (temperature, concentration of movable chemicals, concentration of movable suffusion particles and so on). In this case one requires setting dependencies of values σ and ρ_p from the factors themselves. The same is connected with the σ_m and ρ_m. However, any dependence known from experiments or the theoretical one can be used as this dependence

III. Filtration Equation Taking into Account Interrelated Chemical and Mechanical Suffusion Processes in the Case of the Non-deformed Soil Framework

In this case, the porous liquid moves and the soil solid particles soil are moving too. As a result, we have a generalized law of Darcy-Gersevanov in the case of taking into consideration osmotic phenomena

$$\mathbf{u}-e\mathbf{v}=-\mathbf{K_h}\left(c,T,s,\sigma\right)\nabla h+\mathbf{F_{osm}}, \qquad (4)$$

where e - is the coefficient of soil porosity; h - is the redundant pressure.

Using the approach of classical theory of soil consolidation, applying the continuity equations (1), (2) and the law (4), we obtain the following equation:

$$\frac{R\gamma a}{1+(R-1)\xi}\frac{\partial h}{\partial t}-(1+e)^2\frac{\partial s}{\partial t}-(1+e)^2\frac{1}{\rho_N}\frac{\partial N}{\partial t}-$$
$$-e\left(\frac{1}{\rho_p}\left(\frac{\partial\rho_p}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial\rho_p}{\partial c}\frac{\partial c}{\partial t}\right)-\frac{1}{\rho_m}\left(\frac{\partial\rho_m}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial\rho_m}{\partial N}\frac{\partial N}{\partial t}\right)\right)= \qquad (5)$$
$$=\left(1+e\right)\left(\nabla\cdot\left(\mathbf{K_h}\left(t,h,T,S,N,c\right)\nabla h\right)-\nabla\cdot\mathbf{F_{osm}}\right).$$

To formulate a complete mathematical model, the obtained equations (3) and (5) should be supplemented by the following: 1) equations of interconnected processes of heat transferring in porous media; 2) mechanical suffosion equation; 3) dependencies of the parameters of the porous liquid and the porous medium on the influence of man-caused factors that is the factors, caused by technological activities of people.

IV. Conclusion

The presented technique allowed to take into consideration the finite number of factors of effect, without changing the essence of the process of derivation of the basic equations. The elements of the proposed technique were used in the works [3-5]. The elements of this technique concerning processes in unsaturated porous media were used in the works [2]. Effects of the suffosion-clogging processes influence on the operation of the bio-plate filters are investigated on the basis of the derived models in the work [5].

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